International Journal of Offshore and Polar Engineering
Vol. 8, No. 2, June 1998 (ISSN 1053-5381)

A Weibull-Stokes Model for the Distribution of Maximum Wave and Crest Heights

Raymond Nerzic
OPTIMER, Montpellier, France

Marc Prevosto
IFREMER, Brest, France

ABSTRACT

This paper describes a model for the distribution of maximum wave and crest heights in a given sea state. The standard Rayleigh and Weibull distributions for wave and crest heights are modified using a third-order Stokes expansion of the wave envelope. The maximum wave or crest height in a record is calculated using the asymptotic Gumbel distribution. It was validated by fitting the model distributions to North Sea measurement data and provided much better predictions of maximum crest and wave heights than standard models. To make up for the fact that directionality and spectral shape are not taken into account in the model, correction terms (specific to the studied site) are introduced and mainly one of them will give the optimized mean wave number to consider in the model. The proposed model is relatively easy to apply and could be an effective tool in determining extreme wave and crest heights for offshore structure design purposes.

INTRODUCTION

Standard models for the distribution of wave and crest heights are useful for computing maximum wave and crest heights, on the basis of significant wave heights, for the purposes of designing offshore structures. However, there can be differences of 10%-20% in the results obtained with different models, which is unsuitable in terms of design criteria. These between-model discrepancies can be explained by the problem of accurately representing wave properties, especially strong and steep waves, in a given sea state. This is mainly due to nonlinearities in the wave kinematics which are not suitably taken into account in the models, nonlinearities which are controlled by directionality, spectral shape and wavelength-to-depth ratio.

In this paper, maximum wave and crest heights computed by several models are compared, followed by a description of an analysis of maximum wave and crest heights measured at the Frigg offshore site in the North Sea. The analysis confirmed a marked effect of wave steepness on maximum crest heights, indicating that nonlinear effects occur in wave processes. Steepness did not have as important an effect on maximum crest to trough wave height, however.

This influence is generally not taken into account in wave and crest height distribution models, apart from those developed by Tayfun (1983) and Vinje (1989), as the nonlinear wave process is considered to be a distorted narrow-banded Gaussian process due to nonlinearity effects, and in the transformed Gaussian models (Longuet-Higgins, 1963), (Marthinsen and Winterstein, 1992) and (Rychlik et al., 1997). A model was thus designed for the distribution of wave and crest heights, which combines a Rayleigh or Weibull distribution (describing wave and crest heights as a first-order Gaussian process) and third-order Stokes nonlinear wave expansion. Hereafter this is termed a Rayleigh-Stokes or Weibull-Stokes model.

Tests were performed by fitting distributions derived from this model, along with a maxima distribution model based on a Gumbel distribution, to measured data obtained at the Frigg site, in order to check its validity in representing maximum wave and crest heights.

This model can be applied in several different ways, depending on the wave height, period and steepness parameters considered in the Stokes expansion. Although the model seemed quite suitable for the case investigated, it has not yet been validated for other sea conditions. These points are discussed at the end of the paper.

STANDARD WAVE AND CREST HEIGHT DISTRIBUTION MODELS

The wave amplitude distribution model based on Rayleigh's distribution is the standard model derived from the initial model proposed by Longuet-Higgins (1952). It is based on the assumption of a linear sea-surface, narrow-banded Gaussian process:

$$Q(x) = \mathrm{Prob}(X > x) = \exp\left(-\left(\frac{x}{\theta}\right)^2\right) \qquad (1)$$

where Q is the exceedance probability ($Q = 1 - F$; F is the probability distribution function; X is the H/H_S ratio or the C/H_S ratio; H is the crest-to-trough height; C is the crest height; H_S is the significant wave height, and $H_S = 4.\ m_0^{1/2}$; m_0 is the zero-order moment of the power spectrum, equal to the variance σ^2; and θ is a scale parameter with a value of $(1/2)^{1/2}$, or 0.707 for wave heights, and of value $(1/8)^{1/2}$, or 0.354 for crests.

Empirical wave-height distributions have also been fitted by a Rayleigh distribution for many applications (especially concerning North Sea and Gulf of Mexico situations), with q parameters in the 0.63-0.70 range. Det Norske Veritas (1991) proposed a parameter $\theta = ((1 - c^2\eta^2)/2)^{1/2}$ for wave-height distributions, where c is a constant (value ≈ 1) and h is a spectral width parameter (value ≈ 0.43), or $\theta = 0.638$, and for the crest distribution a

Received March 6, 1997: revised manuscript received by the editors January 13, 1998. The original version (prior to the final revised manuscript) was presented at the Seventh International Offshore and Polar Engineering Conference (ISOPE-97), Honolulu, USA, May 25-30, 1997.
KEY WORDS: Wave height, wave crest, distribution, extreme, nonlinearity.