

# Runup on a Cylinder Due to Waves and Current: Potential Flow Solution with Fully Nonlinear Boundary Conditions

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## ABSTRACT

The application of a fully nonlinear boundary element method to the simulation of wave-body-current interactions in the time domain is presented. Stream function theory is used to model the incoming flow composed of nonlinear regular waves combined with a uniform current. A time-dependent boundary value problem for the nonlinear diffraction flow is formulated and solved using a boundary element method based on triangular isoparametric elements. A 4th-order Runge-Kutta method is applied for the time marching. New and original results on the nonlinear influence of wave steepness and current velocity on the wave runup about a bottom-mounted vertical cylinder are presented and discussed. Results include time series as well as harmonic components of the runup.

## INTRODUCTION

Most recently published numerical methods for the solution of the wave-body-current interaction problem were developed within the frame of frequency-domain analysis, with significant contributions including Nossen et al. (1991), Emmerhof and Sclavounos (1992), Teng and Eatock-Taylor (1995) and Malenica et al. (1995), among others. The advantage of this first approach is to provide results of interest such as wave forces and runups on the structures in a relatively straightforward manner. On the other hand, the mathematical formulation is significantly more complicated than with zero current speed, with specific problems such as secularities. There are also a number of practical limitations, such as regular incoming waves and uniform bottom topography only. Finally, the perturbation expansion of boundary conditions with respect to wave steepness and current speed limits the analysis to linear or weakly nonlinear phenomena, and to the author's knowledge, only linearized formulations have been published to date.

In these conditions, as for a number of other problems (see e.g. Ferrant, 1998), time-domain analysis represents a very attractive alternative. Using a time-domain Rankine panel method, it is theoretically possible to implement any level of boundary condition approximation, from linearized conditions to fully nonlinear ones, and there is no limitation on the geometry. Of course, due to their computational demand which may be very important, the convergence of the numerical models, their stability and accuracy have not yet been sufficiently studied. Generally speaking, there is a remaining lack of confidence in this class of numerical models which will undoubtedly progressively disappear with their development and validation. Applications of time-domain analysis to wave-current interaction problems are still scarce—see for example Kim and Kim (1995), or Büchmann et al. (1997)—and have long been restricted to formulations based on perturbation expan-

sions of free surface conditions. In Kim and Kim (1995), the problem was developed to 1st order in the wave amplitude parameter  $\varepsilon$  and in the current speed parameter  $\tau$ , while in Büchmann et al., a solution up to 2nd order in  $\varepsilon$  and 1st order in  $\tau$  was proposed. In Ferrant (1997), a fully nonlinear potential flow was assumed, but only preliminary results were available. In this paper, we present some results of the application of fully nonlinear time-domain analysis to the wave-current interaction problem in the presence of a 3-dimensional body. The incoming flow, including regular waves and current, is modeled using a stream function theory (Rienecker and Fenton, 1981), and the problem is formulated in terms of the nonlinear perturbation induced in the incident flow by the body, using a formulation initially presented in Ferrant (1996), and further developed in Ferrant (1998), for the capture of higher-order diffraction effects in the time-domain. Simulations with the proposed nonlinear model were run for a given geometry and incoming wave length. Nonlinear influences of the wave steepness and current strength on the wave runup are highlighted by a systematic investigation of these parameters. New results, including time series, maxima and harmonic coefficients of the nonlinear wave runup on a vertical bottom-mounted cylinder, are presented and commented upon.

## THEORETICAL FORMULATION

### Semi-Lagrangian Formulation

We consider a 3-dimensional fluid domain ( $D$ ), bounded by a free surface  $F$  and a set of  $N$  solid boundaries  $S_j$ . These boundaries include the surface of a fixed offshore structure, as well as the sea floor at finite distance. The domain is of infinite extent in the horizontal directions. The fluid flow problem is formulated in the frame of potential flow theory. The fluid velocity inside the domain thus derives from a scalar potential satisfying Laplace's equation:

$$\vec{\nabla} \Phi(M, t) = \vec{V}(M, t) \quad \text{for } M \in (D) \quad (1)$$

$$\Delta \Phi(M, t) = 0 \quad \text{for } M \in (D) \quad (2)$$

On the free surface, both kinematic and dynamic conditions must

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KEY WORDS: Wave-current, vertical cylinder, diffraction, fully nonlinear, BEM, time domain, stream function.