Estimation of Return Levels of Sea Level Along the Swedish Coast by the Method of 
\[ r \] Largest Annual Maxima

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ABSTRACT
Estimation of so-called return values of environmental extremes, of importance in e.g. ocean and coastal engineering, is a challenging problem. Several approaches have been suggested in the literature. In this work, we examine closer a natural extension of the classical approach of block maxima: the \( r \) largest annual maxima. We investigate the performance on measurements of sea level at three locations along the Swedish coastline, and pay attention to the practical implications of the assumptions of the method. Return levels are estimated for various return periods along with related confidence intervals and compared with the conventional block-maxima approach. We find that the upper bound of the confidence intervals is considerably reduced compared to the traditional methodology, in particular for higher return periods.

KEY WORDS: return levels; sea level; coastal engineering; GEV distribution.

INTRODUCTION
In coastal engineering, there is a need of analysis of environmental extremes, not the least when considering risk analysis involving e.g. dams and other infrastructure. A key notion, used as a measure of risk, is the \( T \)-year return level for the quantity of interest (for a recent review, see Volpi, 2019). From a statistical point of view, such levels are estimated from data and several methods are well established, e.g. the conventional method of block maxima or the Peaks Over Thresholds (POT) approach. However, methodology is under constant development, for instance facing the challenges by climate change, implying nonstationary models. For example, Bayesian frameworks have proven to be successful approaches (Fawcett and Walshaw, 2016).

A common issue when estimating return levels is that the uncertainties of the estimates grow larger with increasing return period. The uncertainty is typically presented in the form of a confidence interval, computed by the so-called delta method or by profile-likelihood. In addition, in any extreme-value analysis, the limited amount of data implies problems. The method of block maxima is often considered surprisingly robust, but suffers from that much information in data is not used. The POT methodology, on the other hand, makes more use of data, but the choice of threshold is non-trivial, and some clustering algorithm is needed to cope with possible dependencies.

In this paper, we explore the methodology of the \( r \) largest order statistics (Smith, 1986; Coles, 2001). This method might deserve to be used more frequently than is found in the literature, for instance as an intermediate step between the basic block-maxima approach and more sophisticated Bayesian analyses. Applications are found in the literature for wave height (Soares and Scotto, 2004), wind speed (An and Pandey, 2007), sea level (Hamdi et al, 2014) and temperature (Busababodhin et al. 2021). Within the statistical software R, there is a package, eva, where this approach is implemented (Bader and Yan, 2020).

We investigate the outcomes of the method when applied to three measurement stations of sea level along the Swedish coastline (located close to nuclear plants). More precisely, the widths of confidence intervals for various return periods \( T \) are analysed, as resulting from profile-likelihood approach (implementation in the eva package). Moreover, the influence of the shape parameter in the GEV distribution is studied. The sign of that parameter may have implications for the interpretations (Rydén, 2022).

This paper is organized as follows. First, a review of extreme-value analysis is given, in particular the methodology with \( r \) largest maxima. Next, the three data sets analyzed in this study are presented. Before presenting the very results with estimated return levels for various periods and their associated confidence intervals, comparing with the conventional block-maxima approach, we discuss strategies to handle model assumptions, e.g. independence. Finally, a summary and concluding discussion is provided.

METHODOLOGY
The GEV Distribution
Statistical extreme-value analysis can be said to concern the tails of distributions. The common approach, with roots to Gumbel (1958), is to fit a generalized extreme-value (GEV) distribution to a sample of independent annual maxima (“block maxima”, where a block is one