Determinant of Constitutive Properties of Sand and Clay from Expansion of Spherical Cavities

Vincenzo Silvestri and Claudette Tabib
Department of Civil, Geological and Mining Engineering, Polytechnique Montréal
Montreal, Quebec, Canada

ABSTRACT

This paper presents a method to obtain the constitutive properties of sand and clay from the pressure-expansion curves of spherical cavities. The theory applies to the interpretation of static cone penetration tests, which are often modelled as spherical expansion processes. For sand, the expansion is considered to take place under drained conditions and Rowe’s dilatancy theory is assumed to be valid. For clay, the only assumption that is made is that the expansion takes place under undrained conditions. The theory is applied to the determination of sand properties from a series of experimental spherical expansion tests reported in the literature.

KEY WORDS: Spherical cavities; expansion tests; constitutive properties; sand; clay.

INTRODUCTION

In situ cylindrical expansion tests, such as pressuremeter and dilatometer tests, are often carried out to obtain soil properties for design purposes. For undrained clay response, exact interpretation procedures were independently derived by Baguelin et al. (1972), Ladanyi (1972) and Palmer (1972). For sand, however, there exists various approaches based upon different methods of analysis, i.e. elastic-plastic response with or without volume change, application of Rowe’s dilatancy theory, critical state theory, state parameter approach, and consideration of experimental stress-strain curves (Gibson and Anderson, 1961; Ladanyi, 1961; Vesic, 1972; Hughes et al., 1977; Robertson and Hughes, 1986; Manassero, 1989; Collins et al., 1992; Fahey and Carter, 1993; Yu, 1994; Ladanyi and Foriero, 1998; Wood, 2007; Mo and Yu, 2018).

Another in-situ test is the quasi-static cone penetration test. Recent interpretation procedures regarding this test have taken two complementary directions: the spherical cavity expansion approach (See, for example, Vesic, 1972; Collins et al., 1992; Ladanyi and Foriero, 1998) and the fluid flow approach (See, for instance, Baligh, 1985; Teh and Houlsby, 1991; Yu et al., 2000). While there has been considerable success in modelling clay response using both these approaches, modelling of sand response still constitutes a difficult problem.

The present paper addresses the expansion of spherical cavities both in sand (drained case) and in clay (undrained case), and shows that it is possible to obtain constitutive properties by making a minimum number of assumptions regarding materials’ response. For sands, it is necessary to know the initial isotropic effective pressure at rest and to invoke the validity of Rowe’s stress-dilatancy theory (Rowe, 1962). As for clay, it is only sufficient to require that the expansion takes place under undrained conditions to arrive at an exact solution.

The theory is applied to the interpretation of a series of spherical expansion tests in Ottawa sand reported by Ladanyi and Roy (1987). The relative density I_n of the sand varied between 0.21 and 0.91.

THEORETICAL DEVELOPMENT

A spherical coordinate system (r, θ, ψ) is retained for the analysis, and tensile stresses and strains are considered positive. The principal stresses are σ_r, σ_θ, σ_ψ. The corresponding infinitesimal principal strains are ε_r, ε_θ, ε_ψ. Because the spherical expansion takes place under an initial isotropic pressure p_0, then σ_0 = σ_ψ and ε_0 = ε_v for symmetry. Consequently, the infinitesimal volumetric and principal shear strains, ε_v and γ, are given respectively by:

ε_v = ε_r + 2ε_θ  \hspace{1cm} (1a)

In addition, because large strains are involved in the expansion process, a finite strain approach is used, and the corresponding finite strains are:

ε_0 = ln(1 + ε_r)  \hspace{1cm} (2a)

ε_ψ = ln(1 + ε_0)  \hspace{1cm} (2b)

ε_v = ε_r + 2ε_0  \hspace{1cm} (2c)

γ = ε_0 - ε_r = 3ε_0 - ε_v  \hspace{1cm} (1b)

and

γ = ε_0 - ε_r = 3ε_0 - ε_v  \hspace{1cm} (2d)