Shallow Water Wave Modeling and Focusing Using Recurrent Neural Network

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ABSTRACT

An equivalent model based on recurrent neural network (RNN) is developed in this paper to simulate the propagation of shallow water waves over a varying bathymetry. Shallow water wave theory is applied to establish the mapping between water wave mechanics and an RNN. The resulting RNN model is trained to optimize the varying bathymetry that enables wave energy focusing, using the specified RNN outputs and loss function. We show that the trained RNN model successfully produces different varying bathymetries that are optimized to reflect and diffract incident wave energy to various selected locations.

KEY WORDS: Machine learning; recurrent neural network; varying bathymetry; shallow water wave; wave energy focusing.

INTRODUCTION

Machine learning has made significant progress in many application areas. The most notable examples are application of convolutional neural network (CNN) to computer vision (Russakovsky et al., 2015) and application of RNN to natural language processing (Sutskever et al., 2014). CNN is capable of capturing spatial correlations, which is particularly suitable to process data having spatial characteristics. On the other hand, RNN is capable of capturing temporal/sequential correlations, which makes RNN preferable to signal and language processing. Waves are extremely common phenomena in many physical fields, whose mathematical models are represented by partial differential equations that includes both spatial and temporal derivatives to describe evolution of a physical field over time and space. Hughes and Williamson at el. (2019) study the equivalence between wave dynamics and an RNN, and identify a mapping between the dynamics of sound waves and the computation of RNNs.

This work is inspired by the ground-breaking research of Hughes and Williamson but with a completely different objective. The purpose of Hughes and Williamson’s work is to develop a type of analog machine learning hardware that is faster and more energy efficient than the digital counterparts, while we aim to use machine learning techniques for engineering design: to model, to control, and to optimize the propagation of shallow water waves over a varying bathymetry.

Shallow water wave like many other types of ocean wave is a kind of surface wave, whose dispersion relationship can be simplified and whose celerity is only dependent on water depth and gravitational acceleration. Considering water surface elevation as the primary variable, propagation of shallow water waves can be described by a standard two-dimensional (2D) hyperbolic wave equation if gradient of the bottom is neglected. On the basis of the mild-slope assumption that the relative change of bathymetry over a wavelength is small, Berkhoff (1972) derive the original mild-slope equation in an elliptic form, in which gradient of the bottom is considered in the bottom boundary condition. Smith and Sprinks (1975), Booij (1981), Copeland (1985), and Li (1994) develop the mild-slope equation in time-dependent hyperbolic forms that are similar to standard wave equations. Chamberlain and Porter (1995) and Porter and Staziker (1995) derive the modified mild-slope equation to improve accuracy, in which the square of bottom gradient and the second derivative of bathymetry are considered. Suh et al. (1997) and Lee et al. (2003) derive the time-dependent hyperbolic forms of the modified mild-slope equation and apply to random waves over varying bathymetry.

The shallow water wave theory and the resulting hyperbolic equation are used in this paper to develop an equivalent RNN model. The forward marching in time of this RNN is equivalent to compute the propagation of shallow water waves. The outputs of the RNN model and loss function are specifically selected to represent the design objective, and used to compute training errors. The bathymetry is iteratively updated between training epochs using the error backpropagation through time algorithm. Using the proposed method, we are able to combine the propagation modeling and bathymetry optimization together as the training process of the equivalent RNN model. Different design objectives can be achieved through various training setups for the equivalent RNN.

In the remainder of this paper, we first briefly review the work of Hughes and Williamson, introduce the hyperbolic equations of shallow water wave that can be used to develop the equivalent RNN models, then present the training setup and the training results, and finally discuss the key findings and future improvements.

WAVE DYNAMICS AND EQUIVALENT RNN

The operation of an RNN and how it relates wave dynamics are briefly introduced in this section. Readers can refer to Hughes and Williamson et al. (2019) for more details.