A High-order Numerical Method for Compressible Two-phase Flow in General Curvilinear Coordinates

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ABSTRACT

A numerical strategy of the high-order numerical scheme is adopted to simulate compressible two-phase flow on general curvilinear grids. High-order numerical methods have been extensively employed for the simulation of compressible two-phase flow in the cartesian coordinate system. However, it is more difficult to maintain high accuracy for the numerical methods in general curvilinear coordinates. The numerical errors of numerical schemes on curvilinear grids are easy to hide small scales, which may lead to the inaccurate results or even failures of the simulation. Therefore, the geometric conservation law is used to implement the high-order numerical method on general curvilinear grids. The key idea of this numerical strategy is to offset the geometrically induced errors by discretization of the metric invariants appropriately. For the numerical simulation of compressible two-phase flow, the overestimated quasi-conservative form is used for the numerical strategy, which can maintain the equilibriums of velocity, pressure, and temperature. With this technique, the simulation of compressible two-phase fluid can achieve high accuracy in general curvilinear coordinates. The effectiveness of this strategy is validated by some benchmark test cases. From the perspective of simulation accuracy and efficiency, the high order numerical method on general curvilinear grids s is more promising for the simulation of actual complex compressible multi-phase flows.

KEY WORDS: Two-phase flows; geometric conservation law; quasi-conservative form.

INTRODUCTION

The numerical simulation of two-phase flow has become an interesting and challenging branch of fluid mechanics. As a complex flow phenomenon, it may occur in various forms, such as a flow that changes from a pure liquid to a vapor due to external heating, a separated flow. The mainly feature of compressible two-phase flow is that for materials undergoing phase change, the speed of sound will change dramatically. This necessarily induces compressibility into the problem. Considering the compressible flow conditions of two-phase flow, there are many complex flow phenomena including shock waves and contact surfaces. Therefore, the impact of compressibility must be considered in the design of the numerical algorithm. This requires a numerical strategy with high-order accuracy and good ability to capture discontinuities. The WENO was proposed by Jiang, G. S., and Shu, C. W. (1996), which has a good ability for capturing jumps as well as high accuracy for simulating shock wave. However, the mixed Upwind/Central WENO scheme (Wang, 2018) are developed, which avoid the grid deformation or explicit interface reconstruction. For the compressible two-phase flow, the phase changes are not instantaneous, and the multi-phase flow system will not necessarily be in phase equilibrium. Since the balance of velocity and pressure on mixing the gases in the vicinity of the interface is broken, it may lead to numerical oscillation and bring difficulties to the numerical simulation. Many scholars (Abgrall, R., 1996; Abgrall and Karni, 1996; Johnsen E, and Ham F, 2012) were discussed for this problem. Researches indicate that the conservative form of almost all equations combined with the non-conservative form of the mass fraction (or energy) equation can effectively eliminate the oscillation at the interface. This partially non-conservative form is called “quasi-conservative form” (Abgrall and Karni, 1996). Nonomura et.al (2017) propose that the characteristic-interpolation-based finite difference WENO scheme with quasi-conservative form is used to simulate compressible multicomponent flow, which can keep the balance of velocity, pressure, and temperature at the interface.

In computational fluid dynamics (CFD), it is well known that compared with the finite volume scheme, the finite difference scheme is computationally more efficient and easier to obtain high-order accuracy. Thus, a series of high-precision methods based on finite difference methods have emerged such as Discontinuous Galerkin (DG) method, Spectral Difference Method (SDM), etc. However, Nonomura et.al (2010) analyzed the errors of WENO and WCNS in the application of complex grids respectively, and found that the WENO has poor computational stability and is difficult to apply to the calculation of complex grids. Subsequently, Nonomura (2010) and Jiang (2014) proposed the new numerical strategy for WENO combined with geometric conservation laws (GCL) to enhance the adaptability to complex grids. Zhang et al. (1993) pointed out that the GCL consists of two parts: the volume conservation law (VCL) and the surface conservation law (SCL). VCL is mainly