Structures Stability Based on Path Integral Solution under Random Beam Waves

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ABSTRACT

The stability against capsizing in rough seas is one of the most fundamental requirements considered by naval architects. In this study, Markov process theory is used to simulate the large amplitude rolling motion of ships and offshore platforms with random characteristics and strong nonlinearity. Then, Fokker-Planck equation is used to study the probability density distribution of the process. Because the exact solution of Fokker-Planck equation is difficult to solve, the approximate results are calculated by the path integral solution. The innovation of this paper is to apply this method to the stability analysis of offshore platforms, and the effects of heeling angle on structures stability are discussed.

KEY WORDS: Ship capsizing; offshore platform; Fokker-Planck equation; transition probability density; path integral solution.

INTRODUCTION

The stability of a ship under random wave excitation is directly related to the safety of lives and properties at sea. What’s more, the stability of an offshore platform is the key to ensuring its deep-sea operations can be carried out smoothly. The large amplitude rolling motion is the main cause of capsizing accidents. However, a thorough understanding of capsizing problem in rough waves has not been fully realized. The obstacles mainly come from two areas: firstly, the nonlinearity of the large amplitude rolling motion; secondly, the randomness of wave excitation. The fact that many intact vessels and offshore platforms were lost in rough seas confirms the shortcomings of the current criteria. Therefore, the problems of rolling in stochastic waves should be investigated in stability analysis, and consequently new stability criteria should be established.

Table 1. Statistics of capsizing accidents in recent years

<table>
<thead>
<tr>
<th>Time</th>
<th>Place</th>
<th>Capsizing Accidents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jun.01, 2015</td>
<td>Hubei, China</td>
<td>Luxury Cruise</td>
</tr>
<tr>
<td>Apr.28, 2017</td>
<td>Andhra Pradesh, India</td>
<td>Passenger Ship</td>
</tr>
<tr>
<td>Apr.23, 2018</td>
<td>Congo-Kinshasa</td>
<td>Passenger Ship</td>
</tr>
</tbody>
</table>

In recent years, scholars at home and abroad have analyzed and studied the capsizing of ships under random waves and nonlinear random problems. First, Nekrsov (1978) started to apply Markov process to the study of ship's nonlinear rolling probability model. In the study of nonlinear stochastic theory, Wehner and Wolfer (1987) proved the feasibility of the path integral solution method by comparing the discrete results of a one-dimensional FPK equation with the analytical solutions. Naess and Johnsen (1993) studied the slow drift extreme response of marine structures using path integral solution based on cubic B-spline interpolation. Later, Cai, Yu and Lin (1994) adopted the drift coefficient and diffusion coefficient of ITO equation to study the rolling problem of ships. Yim and Lin (2001) used the path integral solution to solve the FPK equation in the study of ship rolling motion, and obtained the probability density function of ship rolling motion. Huang and Zhu (2001) took the time before capsizing as a reference, and applied Markov process theory and the first pass theory to study the probability density function and duration of ship rolling before capsizing. Chai, Fan and Huang (2013) analyzed the influences of strength of external excitation, rolling damping and nonlinear righting moment to the transition probability density of the ship rolling by path integration method.

Considering the nonlinear rolling damping, nonlinear restoring moment and random wave excitation, the random nonlinear differential equation of rolling is established. It seems that time domain method is an appropriate way in handling such nonlinear problems. What’s more, considering the randomness of wave excitation, the stochastic processes theory can be used for investigating this random event. The response of a dynamic system, whose random excitation (Huang and Fan, 2005) is expressed as white noise process, is a Markov process. Under the theory of Markov process, the probability density of the dynamic system response (such as the transition probability density of ship rolling motion) can be obtained by solving Fokker-Planck equation. However, analytical solutions are only possible for a very limited class of systems. The path integral (PI) method has been proved to be an effective numerical method to solve the Fokker-Planck equation with high accuracy.

In this paper, the two-dimensional Fokker-Planck equation in time domain is solved by using the path integral (PI) method. Firstly, by using white noise as input, a duffing oscillator with known exact solution is used to validate the reasonability of the path integral (PI) method before