

## **The Effect of Axial Extensibility on Three-Dimensional Behavior of Tensioned Pipes/Risers Transporting Fluid**

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### ABSTRACT

This paper presents the effect of axial extensibility on three-dimensional behavior of the tensioned pipes/risers. The large strain model formulation is developed by variational approach based on the elastica theory and the work-energy principle. The finite element method is used to find the nonlinear static configuration and large static strain. The natural frequencies and mode shapes are calculated by using the assumed-modes finite element method. The pipes/risers perform tensioned beam behavior when the bending stiffness dominates, and the stability of the pipes/risers reduces as the axial extensibility increases. On the contrary, the pipes/risers perform the tensioned cable behavior when the axial stiffness dominates, and the stability of the pipes/risers increases as the axial extensibility increases.

**KEY WORDS:** Three-dimensional, Large strain, Tensioned pipes/risers, Axial extensibility, Transporting fluid.

### INTRODUCTION

The tensioned pipes/risers have been applied in various fields of engineering industry. From the literature, several research works such as Skop and Clark (1972), Fang and Lyon (1996), Lee and Mote (1997), Öz and Boyaci (2000), Öz (2001), and Öz and Evrensel (2002) have employed the small displacement theory to study the tensioned pipes behaviors. The previous investigations could predict precisely the linear behavior of the tensioned pipes, which are encountered in applications of the pipelines for conveying gas, oil, water, dangerous liquids in chemical plant, and cooling water in nuclear power plant.

In offshore applications, the tensioned pipes/risers are used as a linkage between the well bore and the floating vessel. Because of extreme ocean environments, the tensioned pipes/risers applied in deep water are usually experiencing the large displacement and large deformation. Consequently, the linear behavior with small deformation which was reported in previous works may be no longer valid. This paper is motivated to investigate the behavior of three-dimensional tensioned pipes/risers with large displacement and large deformation. The emphasis is on the point that variation of the axial extensibility

parameters of the pipes/risers could modify characteristic of nonlinear behavior of the pipes/risers. The effect of the pipes/risers' extensibility on static behavior and dynamic stability of the three-dimensional pipes/risers are illuminated in this work.

To undertake the effect of high extensibility of the three-dimensional pipes/risers, the mathematical model formulation based on the work-energy principles and the extensible elastica theory (Chucheeepsakul et al., 2003; Athisakul et al., 2003) is adopted. The hybrid finite element method based on the principle of stationary potential is used for nonlinear static analysis. For dynamic analysis, the assumed-modes finite element method based on the Rayleigh-Ritz method is employed. Numerical studies are carried out to demonstrate the effect of axial extensibility on large displacements, natural frequencies, and natural mode shapes of three-dimensional pipes/risers.

### VARIATIONAL FORMULATION

#### Internal Virtual Work

Based on the extensible elastica theory (Athisakul et al., 2003), the virtual strain energy of tensioned pipes/risers is

$$\delta U = \int_y [N_a \delta s' + B \kappa \delta \theta' + C \tau \delta \psi' + C \tau \delta \psi'] dy \quad (1)$$

where  $N_a$  is the apparent tension,  $B = EI_{ps}(1 + \varepsilon_d)$  is the bending rigidity,  $C = GJ_{ps}(1 + \varepsilon_d)$  is torsion rigidity,  $\psi$  is the twisting angle,  $s$  is the arc-length,  $\varepsilon_d$  is the dynamic strain,  $I_p$  is the moment of inertia of the pipes/risers,  $J_p$  is the polar moment of inertia of the pipes/risers. According to the differential geometry of space curve, the differential arc-length of the pipes/risers is

$$s' = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{(x'_s + u'_d)^2 + (y'_s + v'_d)^2 + (z'_s + w'_d)^2} \quad (2)$$

where  $u_d, v_d, w_d$  are the dynamic displacements of the pipes/risers.