Variational Principles Related to Motions of a Floating Elastic Plate Considering Wave Radiation Condition

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ABSTRACT

Many analytical methods have been proposed to calculate hydroelastic responses of a very large pontoon-type floating structure in waves. In this paper, variational principles considering wave radiation condition at infinity related to motions of a plate in waves, which are very important in calculations of the elastic response of the pontoon-type floating structure, are discussed. First, Sommerfeld radiation condition at infinity is extended to treat a case with an incident wave. Second, four kinds of variational principles related to motions of the elastic floating plate on a water surface considering the incident and radiated waves are proposed and clarified the mutual relationship of these variational principles. Third, numerical results for elastic response of a floating plate of rectangular and L-shaped plan geometry in waves, which are obtained by using these proposed variational principles, are shown.

KEY WORDS: Variational principle; Elastic plate; Wave-induced elastic motion; Wave radiation condition; Hamilton; Dirichlet; Kelvin.

INTRODUCTION

A Pontoon-type VLFS is one of the typical structural types of very large floating structures (VLFS). Various numerical methods have been proposed to predict the hydroelastic response of this structure in waves (Watanabe (2004); Chen (2006)). These methods are classified into the modal expansion method and the direct method. These analyses are carried out in the frequency domain or in the time domain. Finite element method is used for the structure in order to analyze actual complicated floating structure (Seto et al. (1998); Utsunomiya et al. (2002)).

In relation to the elastic response of Pontoon-type VLFS in waves, four kinds of variational principles related to elastic motions of such a floating plate were derived and made clear the mutual relationship of them (Isshiki (2000), Isshiki and Nagata(2001)). However, in these variational principles, wave radiation condition at infinity was not included and the normal velocity on a vertical cylinder surface of finite size surrounding the plate was designated.

In this paper, four kinds of variational priciples related to motions of such a floating plate, which are very important in calculations for motions of an elastic plate in waves, are derived considering an incident wave and radiated waves from the plate and the mutual relationship of them is made clear.

First, Hamilton-Kelvin's principle, which is one of these variational principles for motions of an elastic plate in waves, is obtained by combining Hamilton's principle related to motions of a plate and Kelvin's Principle related to fluid motion. In this principle, kinematic conditions in motions of plate and water are constraint conditions, and mechanical conditions in motions of plate and water are natural conditions. In this variational principle, arguments for fluid motion are the velocity vector of water and the water elevation. The use of velocity potential is more convenient than that of velocity vector in solving the fluid motion in water wave. Therefore, Hamilton-Dirichet's principle is obtained by transforming the velocity vector in Hamilton-Kelvin's principle into the velocity potential. In this principle, kinematic condition in motions of plate and mechanical condition in motion of water are constraint conditions. Mechanical condition in motions of plate and kinematic condition in motion of water are natural condition. Further, if the solution of fluid motion is obtained by some method, Modified Hamilton-Dirichlet's principle-1 is obtained by constraining the natural condition on motions of water in Hamilton-Dirichet's principle. By eliminating the vertical displacement of the plate in Hamilton-Dirichet's principle by using kinematic conditions between the plate and the water, Modified Hamilton-Dirichlet's principle-2 in which motions of plate and water are expressed only by velocity potential, is also obtained.

As application examples of these variational principles, numerical results for elastic response of a floating plate of rectangular and L-shaped plan geometry in waves, which are obtained by using Modified Hamilton-Dirichlet's principle-1 and Modified Hamilton-Dirichlet's principle-2, are shown

FUNDAMENTAL EQUATIONS FOR MOTIONS OF FLOATING PLATE BY INCIDENT WAVES

We shall consider an elastic floating plate on water as shown in Fig.1. The horizontal shape of the plate and water depth is arbitrary. When there is an incident sinusoidal plane wave of frequency ω, amplitude ς_0 and incident angle α with respect to x-axis, the plate and