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Numerical Calculation of Multidirectional Wave Run-up in Pile Group by a FEM Boussinesq Model

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ABSTRACT

A FEM model with unstructured triangular elements based on the modified Boussinesq equations is developed in this paper. A local coordinate system at the reflecting boundary is introduced to improve the treatment of the oblique waves on the reflecting boundaries. The Adams-Bashforth-Moulton predictor-corrector scheme is used for time integration. The numerical model was used for the simulation for the multidirectional wave propagation through a pile group. The effects of the wave directionality on the wave run-up in the group are numerically investigated.

KEY WORDS: multidirectional wave; FEM; Boussinesq equations; pile group; run-up.

INTRODUCTION

Boussinesq type equations which include also time dependence, weak nonlinearity and dispersion provide a means for studying wave propagation over a slowly varying bathymetry. The first such set of equations for variable water depth was derived by Peregrine(1967), which are referred to as the standard Boussinesq equations. To extend the standard equations to be adapted for deeper water, many modified forms of Boussinesq type equations are given, such as Madsen et al. (1992), Nwogu (1993), Beji and Nadaoka (1996) and so on. To simulate the wave transformation numerically, many numerical models had been established. But most of the models based on the modified forms of Boussinesq type equations are solved by the finite difference method which is easy to use, but is not versatile enough to deal with irregular boundaries. Based on Beji and Nadaoka's equations, we develop a FEM model with unstructured triangular elements. The internal wave maker embedded in an unstructured mesh is used so wave energy which is reflected from the structure would pass through the wave generation line without any numerical distortion. To improve the treatment of the oblique waves on the reflected boundaries at the points of the boundary where the orientation of the boundary segment does not coincide with the global Cartesian axis, a local coordinate system to rotate the Cartesian coordinate to the new (n, T) coordinate system, in which n is aligned with the outward normal and T is the tangent at the boundary node. Typical cases are employed to validate the developed numerical model. Some calculated results show that the model is capable of giving satisfactory predictions and accuracy because of the improved treatment of the oblique boundaries.

In addition, real waves are multidirectional, with the wave energy distributed over a wide range of frequencies and directions. The wave directional spreading has a definite effect on the wave transformation and action on structures. In this paper, the numerical model was used for the simulation of the multi-directional wave propagation through several pile groups. The effects of the wave directionality on the wave run-up in the group are numerically investigated.

MATHEMATICAL FORMULATION

The Boussinesq equations derived by Beji and Nadaoka (1996) is:

$$\eta_t + \nabla\cdot\left[(h+\eta)\mathbf{u}\right] = 0 \tag{1}$$

$$\begin{aligned}&\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} + g\nabla\eta = \\ &(1+\beta)\frac{h}{2}\nabla\left[\nabla\cdot(h\mathbf{u}_t)\right] + \beta\frac{gh}{2}\nabla\left[\nabla\cdot(h\nabla\eta)\right] \\ &-(1+\beta)\frac{h^2}{6}\nabla\left[\nabla\cdot(\mathrm{u}_t)\right] - \beta\frac{gh^2}{6}\nabla\left[\nabla^2\eta\right]\end{aligned} \tag{2}$$

where $\mathbf{u}$=(u, v) is the two-dimensional depth-averaged velocity vector, η is the water surface elevation, h=h(x, y) is the water depth, g is the gravitational acceleration, ∇ is the horizontal gradient operator, the subscript t represents the partial differentiation with respect to time, β is a constant and here is 1/5.

Grouping the time derivatives together, Eqs. (1) and (2) can be re-written in scalar form as follows:

$$\eta_t + \frac{\partial}{\partial x}\left[(h+\eta)u\right] + \frac{\partial}{\partial y}\left[(h+\eta)v\right] = 0 \tag{3}$$

$$\begin{aligned}&p_t + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial\eta}{\partial x} - \frac{\beta gh}{2}\frac{\partial}{\partial x}\left[\frac{\partial(hc)}{\partial x} + \frac{\partial(hd)}{\partial y}\right] \\ &\quad + \frac{\beta gh^2}{6}\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 d}{\partial x\partial y}\right) = 0\end{aligned} \tag{4}$$