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# The Relation Between Wave Length and Wave Period Distributions in Random Gaussian Waves

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## ABSTRACT

**Wave data often describe the variation of sea elevation with time, at a fixed point. Wave period is the time between successive crossings of the mean level, while wave length is the distance between crossings at a fixed time along a fixed direction. For a deterministic, harmonic wave, the dispersion relation describes the relation between wave period and wave length. For random waves, wave period and wave length, as well as wave amplitude, are random quantities. We study these distributions for Gaussian waves and show how they can be calculated exactly from the general wave frequency spectrum and wave number spectrum, respectively. The results show that the dispersion relation leads to considerable underestimation of short waves for wide band spectra. We also consider the effect of truncation of high frequencies, to illustrate the sensitivity of the wave length distribution to spectrum truncation. Further, we give examples of wave period or wave length and wave amplitude distributions in sea states with unidirectional and directional spectrum.**

## INTRODUCTION

In a random sea, wave characteristics such as wave period and wave length are random quantities. Their distributions can be interpreted as long-run empirical distributions when observing the sea at a single point over a long time interval, for wave period, and over a long distance at a single instant of time, for wave length. Thus, there is no immediate relation between the two distributions, but they can both be calculated from the energy spectrum of the waves. The dispersion relation connects wave length to wave period for each elementary wave thus building up the total wave package, and in waves with very narrow energy spectrum this also gives an approximately correct relation between wave period and wave length distributions in the resulting sea. For not so narrow spectra, however, there is no direct relationship between apparent wave period and wave length, and the use of the dispersion relation to derive the wave length distribution directly from an observed wave period distribution can lead to considerable errors. The object of this paper is to show exactly how different the two distributions are for different types of spectra, and to how much error a direct use of the dispersion relation will lead.

The wave period and wave length distributions in a Gaussian sea are completely determined by the frequency spectrum and the wave number spectrum, respectively. Since the two spectra are directly related to each other, one can therefore calculate the exact wave period and wave length distributions as soon as one of the two spectra is known. Most existing wave data statistics are based on measurements of the sea elevation at a fixed point on the sea, and empirical wave frequency spectra are easily obtained from these data. Empirical wave number spectra are seldom to be found, even if measurements from satellite-borne synthetic aperture radar (SAR) provide the means for estimation of parts of the wave number spectrum, including directional information; see for example Hasselmann et al. (1996).

The theoretical statistical analysis of wave characteristics goes back to the classical work by St. Denis and Pierson (1953) and Longuet-Higgings (1956), which in turn relied on the pioneering work by S.O. Rice (1944, 1945). The exact statistical distributions are in general very difficult to obtain, but for Gaussian sea states, one can calculate the distributions numerically with high accuracy, based on the energy spectrum density function alone by the methods described in Rychlik and Lindgren (1993, 1995). (Note that no Monte Carlo simulations are involved in this method but only numerical integration of the relevant multidimensional integrals.)

In this paper we shall compare the wave length and wave period distributions for a number of Gaussian processes with different spectral width, and in particular we shall investigate the effect of truncation in the common Pierson-Moskowitz frequency spectrum. We shall also illustrate the effect of a directional spectrum on the distribution of the wave length measured in different directions in relation to the main wave direction.

The joint distribution of wave period, wave length and amplitude can also be found by the methods in Rychlik and Lindgren (1993, 1995). We give examples of such distributions for unidirectional and directional waves.

## BASIC NOTATION

We consider first a one-dimensional wave profile along a certain fixed direction, and denote the wave process by $\xi(t,x)$, where $t$ is time and $x$ is the distance from the origin in the chosen direction. We assume the mean level to be zero and define the following wave characteristic variables:

$T$ = half zero crossing wave period (1)
= time from zero down- to up-crossing,

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