A Practical Method for Predicting Sinkage and Trim

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A practical method for computing the sinkage and trim experienced by a ship that advances at constant speed through calm water of large depth and lateral extent is considered. This method is based on a slender-ship approximation that defines the flow about a ship explicitly in terms of the ship speed and hull shape. Thus, the method is particularly simple, robust and efficient. In spite of its simplicity, the method yields predictions of sinkage and trim in reasonable agreement with experimental measurements.

INTRODUCTION

The sinkage and trim experienced by a ship that advances at constant speed through calm water of large depth and lateral extent is considered. A large number of well-established alternative methods for computing steady free-surface flow about a ship have been developed. These methods include semi-analytical theories based on various approximations (thin-ship, slender-ship, 2-D+t theories); potential-flow panel (boundary integral equation) methods that use a Green function (elementary Rankine source, or Havelock source that satisfies the radiation condition and the Michell linearized free-surface boundary condition); and CFD methods that solve the Euler or RANS equations. These alternative calculation methods are reported in a huge body of literature, not reviewed here. Every one of these alternative calculation methods can be used to predict the sinkage and trim experienced by a ship, for example Subramani et al. (2000) and Yang and Löhner (2002).

Many practical applications require the very quick assessment of numerous alternative designs, as is typically considered during preliminary and early design stages. For such practical applications, and for hydrodynamic hull-form optimization, a simple approximate calculation method that is easy and fast to implement, very robust, and highly efficient is useful, if not necessary. Such a method is used in this study. The method is based on the slender-ship flow approximation and the simple free-surface Green function given in Noblesse (1983) and Yang et al. (2004).

SLENDER-SHIP FLOW APPROXIMATION

Steady free-surface potential flow about a ship that advances at constant speed $\mathcal{U}$ through calm water of large depth and lateral extent is now considered. Nondimensional coordinates

$$\mathbf{x} = (x, y, z) = \mathbf{X}/L$$

are defined in terms of a reference length L, typically taken as the ship length. The z axis is vertical and points upward, and the

mean free surface is taken as the plane $z = 0$. The x axis is chosen along the path of the ship and points toward the ship bow. The Froude number F is defined as:

$$F = \mathcal{U}/\sqrt{gL}, \tag{1}$$

where g is the gravitational acceleration.

The flow about the ship is observed from a frame of reference attached to the advancing ship. The flow observed in this moving system of coordinates is steady (independent of time) and given by the sum of a uniform current that opposes the ship speed $\mathcal{U}$ and the (disturbance) flow:

$$\mathbf{U} = (U, V, W) = \mathcal{U}(u, v, w) = \mathcal{U}\mathbf{u}$$

due to the ship.

A simple approximation to the flow velocity $\mathbf{u}$ is used here. This approximation is defined explicitly in terms of the ship speed $\mathcal{U}$ and length L (Froude number F), and the shape of the mean wetted ship hull $\mathcal{H}$. Specifically, the flow velocity $\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v}, \tilde{w})$ at a flow-field point $\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y}, \tilde{z})$ is given by:

$$\tilde{\mathbf{u}}(\tilde{\mathbf{x}}) \approx \int_{\mathcal{H}} d\mathscr{A}(\mathbf{x}) \, n^x(\mathbf{x}) \, \tilde{\nabla} G(\tilde{\mathbf{x}}; \mathbf{x}). \tag{2}$$

In this expression $\mathbf{x} = (x, y, z)$ is a point of the mean wetted ship hull $\mathcal{H}$, $d\mathscr{A}(\mathbf{x})$ stands for the differential element of area of $\mathcal{H}$ at $\mathbf{x}$, and $n^x(\mathbf{x})$ is the x-component of the unit vector $\mathbf{n} = (n^x, n^y, n^z)$ normal to the ship hull $\mathcal{H}$. The unit vector $\mathbf{n}$ normal to $\mathcal{H}$ points outside the ship hull, i.e., into the water. Further, $\tilde{\nabla} = (\partial/\partial_{\tilde{x}}, \partial/\partial_{\tilde{y}}, \partial/\partial_{\tilde{z}})$, and $G = G(\tilde{\mathbf{x}}; \mathbf{x})$ stands for a Green function associated with the Michell free-surface boundary condition:

$$G_z + F^2 G_{xx} = 0 \quad \text{at } z = 0. \tag{3}$$

Eq. 2 corresponds to the slender-ship approximation given in Noblesse (1983). More precisely, this approximation involves a line integral around the ship waterline. However, this waterline integral yields only a small contribution and is then ignored in Eq. 2. The slender-ship approximation (Eq. 2) merely provides an approximation to the flow about a ship. Yet, this approximation has been found useful for many practical applications. In particular, the method provides a practical tool useful for the very quick