

The Scattering and Damping of Ice-coupled Waves

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Ice-coupled waves travelling beneath solid ice sheets experience decay arising from both scattering (reflections by inhomogeneities) and damping (a consequence of the viscoelasticity in the sea ice and energy loss in the water). Scattering and damping have been examined in isolation but rarely together, which is necessary if a model is to simulate reality accurately. This deficiency is addressed here. We describe a model that assimilates these mechanisms, which is used to reproduce waves under 2-dimensional ice sheets of variable thickness. Damping is more significant for lower period waves, and scattering causes a reduction in the observed decay rate.

INTRODUCTION

When surface gravity waves generated by storms enter a region of ice-infested sea, they may propagate under the ice cover as flexural gravity waves (sometimes known as ice-coupled waves). These waves are typically long and low and can travel deep into the icy polar oceans. Their study has been the target for intensive research of late, summarised in reviews by Squire et al. (1995) and Squire (2007). The details of the topics examined are often conveniently categorised by the type of sea ice being considered: either the more fragmented floes of the marginal ice zone, or the continuous cover of the inner ocean. Here we consider the latter.

Early researchers treated the ice as an homogeneous elastic plate, examining problems such as waves incident on the edge of an ice sheet (Fox and Squire, 1994). However, sea ice is heterogeneous and may contain flaws such as cracks, leads and ridges that cause the waves to scatter (so they are partially reflected and transmitted), as modelled by, amongst others, Squire and Dixon (2000 – cracks), Chung and Linton (2005 – leads), and Williams and Squire (2004 – ridges).

However, real ice sheets do not conform to the “flat sheet with inhomogeneities model” and to that end Vaughan et al. (2007) examined the scattering coefficients of long random sheets while Vaughan and Squire (2006) simulated propagation under long transects sampled from real ice data obtained using upward-pointing sonar during submarine voyages under the ice. Problematic in these 2 papers are resonances associated with the lengths of the transects, an arbitrary and artificial length, which become significant when transects of more than several hundred m are considered. In 2 dimensions, when multiple scatters are present, waves exhibit exponential decay (Vaughan and Squire, 2007).

It is well known that sea ice is not perfectly elastic, but most researchers use the Euler-Bernoulli thin plate equation to model the ice sheet (Squire, 2007). Some researchers have accommodated the effect of viscosity; in particular, Squire and Fox (1992) used the plate equation of Robinson and Palmer (1990) to model waves in exceptionally viscous ice. From the experimental results

of Squire and Fox (1992) we anticipate that viscosity will be made evident as an exponential decay of the wave elevation envelope. It is possible that the viscoelastic nature of sea ice will be significant for scattering across long transects, but models that accommodate both scattering and damping due to viscosity are not common in the literature. Such a model is described in this paper.

Here we consider 2 methods for including viscosity. Note that inhomogeneities in ice sheets are usually set between 2 homogeneous semi-infinite sheets. We first examine the consequence of taking the viscosity as zero in the semi-infinite sheets for scattering by a rectangular ridge, and then compare this to the case where the viscosity is constant in all regions. We conclude by examining the damping and scattering in a long transect comprising a piecewise continuous sheet with random thicknesses.

FORMULATION OF PROBLEM

Consider a length of sea ice for which the thickness is a piecewise constant, for solution purposes, placed between 2 semi-infinite sheets of constant thickness. Such an ice sheet is shown schematically in Fig. 1. Each discontinuity in the thickness will result in the partial reflection of an incident wave train, with both the change in mass loading and the rigidity contributing to the nature of the scattering. In order for the solution technique discussed here to work, it is necessary for the ice sheet to have a flat underside, so all variations in thickness must lie on top of the sheet. Squire and Williams (2008) note that this approximation is justified, i.e. the effect of Archimedian draft is not major except at very short periods.

The equations governing the velocity potential (Φ) are:

$$\nabla^2 \Phi(x, z, t) = 0, \quad (1a)$$

$$\rho_w \Phi_t(x, 0, t) + P(x, 0, t) - P_a - \rho_w g \eta(x, t) = 0, \quad (1b)$$

$$\Phi_x(x^+, z, t) - \Phi_x(x^-, z, t) = 0, \quad (1c)$$

$$\Phi(x^+, z, t) - \Phi(x^-, z, t) = 0, \quad (1d)$$

$$\Phi_z(x, 0, t) - \eta_t(x, t) = 0, \quad (1e)$$

$$\Phi_z(x, H, t) = 0, \quad (1f)$$

which are respectively Laplace’s and Bernoulli’s equations, the 2 conditions requiring that the horizontal flow be continuous, the

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