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Wave Scattering by an Infinite Straight-Line Array of Axisymmetric Floes

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The linear and time-harmonic problem of an infinite, equispaced, straight-line array of identical floes is considered. Under the asssumption of axisymmetry, the floes are permitted to vary in thickness through both their upper and lower surfaces. Further, a realistic draught may be incorporated. An approximation technique is implemented, which combines a variational principle and an association of the vertical motion to the mode that supports propagating waves. The resulting equations exist in the horizontal plane and are solved through the use of a Green's function in the free-surface domain and a Fourier expansion in the ice-covered domain. Numerical results are given for example problems that show the influence of the varying thickness and realistic draught.

INTRODUCTION

In the Arctic Basin and Southern Ocean the marginal ice zone (MIZ) comprises a vast number of individual ice floes, as well as regions of quasi-continuous ice cover. These floes are created on a seasonal basis through the freezing of surface water or from the fracture of the larger ice sheets, which is caused primarily through stresses induced by ocean waves. For this reason, the smaller, individual floes often form a barrier between the open ocean and the inner mass of ice. Waves that reach this inner mass will thus need to penetrate the boundary of smaller ice floes, and in doing so they will become attenuated. Occasionally, the band of individual floes and ice cakes may also detach from the rest of the ice pack to create a dissociated belt off the ice edge (Bauer and Martin, 1980).

The ice covering in the MIZ is relatively thin in comparison to its horizontal dimensions, and the standard method for modelling this situation is to use thin-plate theory (Timoshenko and Woinowsky-Krieger, 1959). In this context, the motions of the fluid-ice system, caused by ocean waves, manifest as flexural-gravity waves that travel at the interface between the 2 media. When forced by an incident ocean wave, as well as exhibiting the so-called rigid-body modes, a floe will have an individual response, in which the floe itself flexes and scatters the wave in the surrounding free surface. Notwithstanding this, when a number of floes is present, we must also take account of the interaction of the floes by considering the simultaneous dependence of each floe on the waves scattered by the other floes. So, in addition to the individual response of the floes, for multiple floes there is a net response of the conglomeration to consider as well.

Due to the complexity of the coupling between the ice and fluid, it is often assumed that floes are of a uniform thickness and have zero draught in order to facilitate a solution. However, by using a variational principle and associating the vertical motion with the mode that supports propagating waves, Porter and Porter (2004) were able to correctly derive approximations for 2-dimensional floes of varying thickness and Archimedian draught. This approximation was later extended to a full-linear solution by Bennetts, Biggs and Porter (2007) by including a finite number of the modes that support evanescent waves.

A number of papers has investigated the way in which 3-dimensional ice floes react to and scatter incident waves. It is usual to assume some simplification of the shape of the floe in order that a solution may be found. For instance, Meylan and Squire (1995), Peter, Meylan and Wang (2004) and Andrianov and Hermans (2004) all treat the case of a circular floe of uniform thickness and a zero draught. Other papers have attempted to elaborate on this structure. Meylan (2002), for example, constructs a method based on the numerical evaluation of the in vacuo modes of the plate to generate solutions for a floe of uniform thickness and a zero draught, but where the shape of the floe is arbitrary. Using the multi-mode expansion of the vertical motion noted above, Bennetts, Biggs and Porter (2008) investigated the case of a circular floe of a non-zero draught and axisymmetrically varying thickness.

It is widely recognised, however, that in order to assess the scattering of waves in the MIZ we must consider a model that comprises a large number of individual floes, in which floes interact with one another as well as with the incident wave. To this end, some research has been conducted on multiple floe arrays. Peter and Meylan (2004) extended Meylan (2002) through the use of local coordinate systems and Graf's addition theorem in order to combine the effects of a finite number of individual floes. In this case, the computational sustainability of the method becomes strained as the number of constituent floes increases. For this reason, unphysical periodic problems, involving an infinite number of arrays, have been studied in order to represent a large number of floes whilst keeping the numerics manageable. In one such paper, Peter, Meylan and Linton (2006) build upon the work of Peter and Meylan (2004) and invoke periodicity to solve for a straight-line array of identical ice floes on a fluid domain that stretches to infinity in all lateral directions. Wang, Meylan and Porter (2007) consider the same problem as Peter, Meylan and Linton (2006) but use a periodic Green's function as their method of solution.

In the work outlined in this paper, we also consider a straight-line array of equispaced, identical floes on an infinite fluid surface. However, unlike Peter, Meylan and Linton (2006) and Wang, Meylan and Porter (2007), we allow our floes to vary in thickness

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