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Random Forces on a Slender Vertical Cylinder Given by High Sea Waves Interacting with a Current

Felice Arena* and Alessandra Romolo*
Faculty of Engineering, Università Mediterranea of Reggio Calabria, Reggio Calabria, Italy

This paper investigates the Morison force on a slender vertical cylinder, produced by random wave groups with large waves, either in an undisturbed wave field or for waves superimposed on a uniform current. For this purpose Boccotti's Quasi-Determinism theory is extended to wave-current interaction. Thus, assuming that a very large wave occurs at some fixed time and location for a fixed value of current velocity, the analytical expressions of the free-surface displacement and of the velocity potential are obtained. Finally, it is found that the maximum wave force given by the New Wave model, which is suggested by the API recommendations for the calculation of wave forces of sea waves on a structure, tends to underestimate the maximum total force given by the Quasi-Determinism theory.

INTRODUCTION

The aim of this paper is to evaluate the hydrodynamic forces on a vertical cylindrical structure, given by random sea wave groups with large waves and a uniform current.

The kinematics of wave groups is obtained by extending the Quasi-Determinism theory, formulated by Boccotti (1981, 1982, 1983, 1989, 1997, 2000), to the wave-current interaction. Both formulations of the Quasi-Determinism theory are examined. The first formulation (Boccotti, 1981, 1982) enables us to establish what happens when a very large crest occurs at a fixed point and time in a random wave field; this formulation was also proposed, for time domain, by Tromans et al. (1991), who renamed the theory New Wave. On the other hand, the second formulation (Boccotti, 1983, 1989, 1997, 2000) deals with the highest crest-to-trough waves, and it enables us to obtain the deterministic profile of sea waves, as well as the velocity potential, when an exceptional crest-to-trough wave height occurs in a Gaussian sea state.

First we investigate the wave kinematics in time domain, produced by random wave groups, by applying both the second formulation of the Quasi-Determinism theory and the New Wave model (first formulation of the Quasi-Determinism theory). The free-surface displacements and the wave kinematics are then compared to achieve the largest Morison force (Morison et al., 1950) on a vertical cylinder when a high wave occurs.

Afterwards, by extending the Quasi-Determinism theory to the case of sea wave groups propagating in the presence of a current (following or opposing current), we achieve the horizontal component of both velocity and acceleration. They are functions both of the wave number in the presence of a current related to the angular frequency and to the current velocity for a fixed value of bottom depth (Phillips, 1966), and of the frequency spectrum related to the current velocity (Huang et al., 1972; Hedges et al., 1985).

Finally, the forces on a vertical cylinder due to high waves and current are calculated by applying the Morison equation for fixed values of wave height and current velocity (following or opposing currents).

INTERACTION OF WIND-GENERATED WAVES AND UNIFORM CURRENTS

Theory of Sea States in Presence of a Uniform Current

Following the theory of wind-generated waves given by Longuet-Higgins (1963) and Phillips (1967), a random sea state may be defined as a sum of a very large number, N, of periodic components, with infinitesimal amplitude, a_i, frequencies, ω_i, different from each other, and phase angles, ε_i, uniformly distributed on $(0, 2\pi)$ and stochastically independent of each other.

Consequently, the linear free-surface displacement and the velocity potential are stationary Gaussian random processes and, in particular for long-crested random waves interacting with a uniform current, their expressions are respectively provided by:

$$\eta(y,t) = \sum_{i=1}^{N} a_i \cos(k_{C_i} y - \omega_i t + \varepsilon_i) \tag{1}$$

$$\phi(y,z,t) = uy + g\sum_{i=1}^{N} a_i(\omega_i - uk_{C_i})^{-1} \frac{\cosh[k_{C_i}(d+z)]}{\cosh(k_{C_i} d)} \cdot \sin(k_{C_i} y - \omega_i t + \varepsilon_i) - \frac{1}{2}u^2 t + \frac{1}{\rho}\int_0^t f(t')\,dt' \tag{2}$$

where, in the presence of a current, the wave number, k_{C_i}, is related to the angular frequency, ω_i, and the current velocity, u, for fixed depth, d, by the following expression:

$$k_{C_i} \tanh(k_{C_i} d) = \frac{(\omega_i - uk_{C_i})^2}{g}, \tag{3}$$

which is the *linear dispersion rule* modified by current (Phillips, 1966), valid for every component of a sea state. (Note that for $u = 0$, Eq. 3 gives the classical *linear dispersion rule*.)

In addition, the amplitudes a_i are such as to define a frequency spectrum $S(\omega, u)$ that is a function of the spectrum of wind-generated waves $S(\omega)$ by means of the relation (Hedges et al., 1985):

$$S(\omega,u) = \frac{G(k_0 d)}{\dfrac{u}{(\omega - uk_C)d} + G(k_C d)} S(\omega) \tag{4}$$

*ISOPE Member.

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