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Intrinsic Coordinate Elements for Large Deflections of Offshore Pipelines

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ABSTRACT

A numerical method for solving large deflections of elasticas and offshore pipelines is described. The method is based on finite element analysis using intrinsic coordinates, namely the nodal rotations and the arc length. The element stiffness is independent of element orientation and the displacement vector is expressed in terms of nodal values of cross-sectional rotation. The generation of the vector of integrating coefficients for numerical integration adopts the quadrature method, which is subsequently assembled into a quadrature matrix. The element lengths are embodied in the vector of integrating coefficients and such lengths are not constrained to be uniform. This feature fits in very well with the intrinsic coordinate elements. The transformations that are required from intrinsic to Cartesian coordinates affect only the load vector in the equilibrium equation. Consequently some computational advantages can be expected from the intrinsic coordinates formulation, particularly for large deflection problems.

INTRODUCTION

During installation an offshore pipeline can undergo large deflections with small strain, i.e. the classical elastica problem where axial and shear deformations are neglected. The equations for large deflection are nonlinear and the length of the suspended pipeline during installation is not known a priori. Numerical methods, particularly the finite element method, are often used to solve the problems. The finite element formulation usually refers the load-displacement interaction to a Cartesian coordinate system. In the present finite element formulation, the displacement field is specified in terms of cross-sectional rotation $\Psi(s)$, where s is measured along the deformed axis of an element. This method is explored for a variety of elastica examples, including pipeline problems.

FINITE ELEMENT FORMULATION AND EQUILIBRIUM EQUATION

The present development is based on the assumption that the element is elastic in bending and can undergo large displacements with small strain, i.e. the classical elastica problem with axial and shear deformation neglected (Frisch-Fay, 1962). The displacement field of an element is specified in terms of the cross-sectional rotation $\Psi(s)$, where s is measured along the deformed axis of the element. The coordinates (s, Ψ) are referred herein as the intrinsic coordinates (Wang, 1986).

The strain energy due to bending, U^e of an element of length and flexural rigidity, EI is:

$$U^e = \int_0^l \frac{EI}{2}\left[\frac{d\psi}{ds}\right]^2 ds \tag{1}$$

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The displacement field $\Psi(s)$ is defined by a continuous polynomial, so that the rotations within any element may be interpolated from the nodal values on that element. Nodes are positioned at each end of an element so that equality of nodal values will satisfy interelement compatibility. A 2-node element is therefore the minimum configuration to satisfy compatibility. That element may be called a linear element, since the rotation varies linearly within the element. Additional internal nodes of rotation can also be generated within the element for more accurate representation of the displacement field. In this paper, a 3-node element is called as a quadratic element and a 4-node element is a cubic element.

The polynomial $\Psi(s)$ may be expressed as a Lagrangian interpolation polynomial $\psi(s) = [\beta]\{\bar{\theta}\}$, where $[\beta]$ is the coefficient function and $\{\bar{\theta}\}$ the vector of nodal rotations. Substituting the Lagrangian interpolation polynomial into Eq. 1 gives:

$$U^e \approx \frac{1}{2}\{\bar{\theta}\}^T\left[\int_0^l EI[\beta]^T[\beta]ds\right]\bar{\theta} \approx \frac{1}{2}\{\bar{\theta}\}^T[\bar{K}]\{\bar{\theta}\} \tag{2}$$

where the element matrix $[\bar{K}]$ represents the integral term. The $[\bar{K}]$ matrices, for the linear, quadratic and cubic elements, respectively, are:

$$\frac{EI}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$
$$\frac{EI}{3l}\begin{bmatrix}7 & -8 & 1\\ & 16 & -8\\ sym & & 7\end{bmatrix} \tag{3}$$
$$\frac{EI}{l}\begin{bmatrix}3.7 & -4.725 & 1.35 & -0.325\\ & 10.8 & -7.425 & 1.35\\ & & 10.8 & -4.725\\ sym & & & 3.7\end{bmatrix}$$

The strain energy of an elastica is the sum of the strain energies of the component elements, i.e. $U = \Sigma U^e = 1/2\{\theta\}^T[K]\{\theta\}$, where $\{\theta\}$ and $[K]$ are assembled from the element displacement vectors